

Engineering Maths.

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Q1. $V = \frac{V_0}{\sqrt{r^2 - 1}}$

① $V\sqrt{r^2 - 1} = V_0$
 $\sqrt{r^2 - 1} = \frac{V_0}{V}$ ✓

$r^2 - 1 = \left(\frac{V_0}{V}\right)^2$ ✓

$r^2 = \left(\frac{V_0}{V}\right)^2 + 1$ ✓

$r = \sqrt{\left(\frac{V_0}{V}\right)^2 + 1}$ ✓

5 ✓

Q2. a) $5(3x - 2) - 2(4x - 3) + 25 = 0$
 $15x - 10 - 8x + 6 + 25 = 0$ ✓
 $7x + 21 = 0$ ✓

$7x = -21$ ✓

$x = \frac{-21}{7}$ ✓

$x = -3$ ✓

4 ✓

b) $6x^2 + 13x - 5 = 0$

~~$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$~~
 ~~$= \frac{-13 \pm \sqrt{13^2 - 4 \times 6 \times -5}}{2 \times 6}$~~

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-13 \pm \sqrt{13^2 - 4 \times 6 \times -5}}{2 \times 6}$ ✓

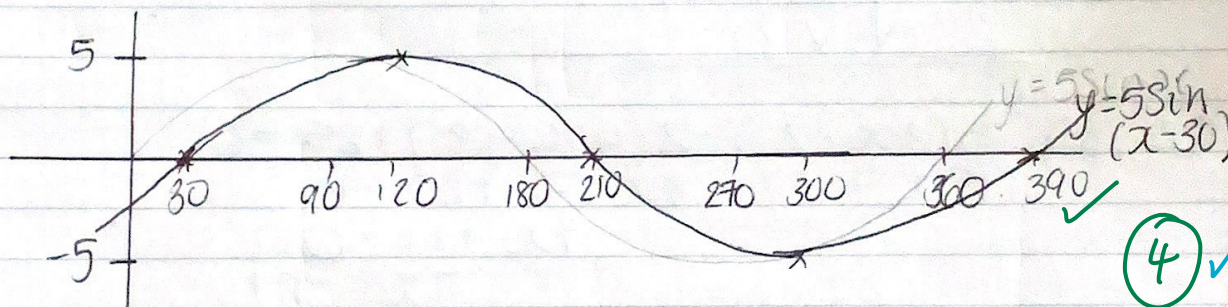
$= \frac{1}{3} \text{ or } = -\frac{5}{2}$ ✓

5 ✓

$$\begin{aligned}
 c) \quad 5 \times 10^{3x+2} &= 215 \\
 10^{3x+2} &= \frac{215}{5} \\
 10^{3x+2} &= 43 \\
 3x+2 &= \log_{10} 43 \\
 3x &= \log_{10} 43 - 2 \\
 x &= \frac{\log_{10} 43 - 2}{3} \\
 x &= -0.122
 \end{aligned}$$

⑥

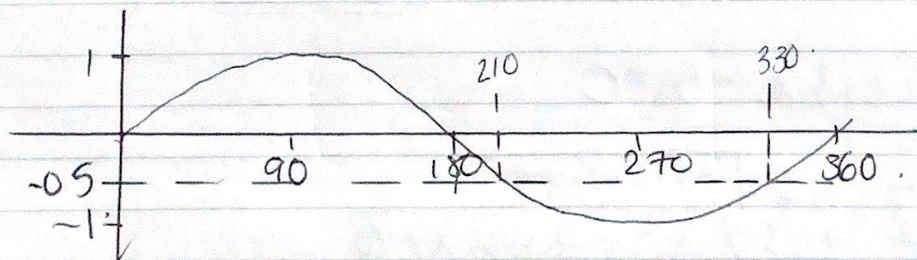
Q3. a) $y = 5 \sin(x - 30^\circ)$



④

b) $\sin 2x = -0.5$

$$\begin{aligned}
 \sin^{-1}(-0.5) \\
 2x &= -30^\circ
 \end{aligned}$$



$$\sin^{-1}(-0.5) = -30^\circ + 360^\circ = 330^\circ$$

$$\begin{aligned}
 2x &= 210^\circ, 330^\circ, 210^\circ + 360^\circ, 330^\circ + 360^\circ \\
 2x &= 210^\circ, 330^\circ, 570^\circ, 690^\circ \\
 x &= 105^\circ, 165^\circ, 285^\circ, 345^\circ
 \end{aligned}$$

⑨

Q4. $z_1 = 3 - j5$ $z_2 = -2 + j4$

a) $z_1 - z_2 = 3 - j5 - (-2 + j4)$
 $= \underline{\underline{5 - j9}}$ ✓

(2) ✓

b) $z_1 z_2$
 $= (3 - j5)(-2 + j4)$
 $= -6 + j12 + j10 - j^2 20$
 $= -6 + j22 - (-1)20$
 $= -6 + j22 + 20$
 $= \underline{\underline{14 + j22}}$ ✓

(4) ✓

c) $\frac{z_1}{z_2} = \frac{3 - j5}{-2 + j4} \times \frac{-2 - j4}{-2 - j4}$ ✓

$= \frac{-6 - j12 + j10 + j^2 20}{4 + j8 - j8 - j^2 16}$ ✓

$= \frac{-6 - j2 + (-1)20}{4 - (-1)16}$ ✓

$= \frac{-6 - j2 - 20}{4 + 16}$ ✓

$= \frac{-26 - j2}{20}$ ✓

$= \frac{-26}{20} - \frac{j2}{20}$ ✓

$= \underline{\underline{\frac{-13}{10} - \frac{j}{10}}}$ ✓

(7) ✓

Q5. $A = \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 0 \\ -2 & 6 \end{bmatrix}$

a) $3A - 2B = 3 \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} - 2 \begin{bmatrix} 5 & 0 \\ -2 & 6 \end{bmatrix}$
 $= \begin{bmatrix} 12 & -3 \\ 6 & -9 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -4 & 12 \end{bmatrix}$
 $= \begin{bmatrix} 2 & -3 \\ 10 & -21 \end{bmatrix}$ ✓

③ ✓

b) $AB = \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ -2 & 6 \end{bmatrix}$
 $= \begin{bmatrix} 4 \times 5 + -1 \times -2 & 4 \times 0 + -1 \times 6 \\ 2 \times 5 + -3 \times -2 & 2 \times 0 + -3 \times 6 \end{bmatrix}$
 $= \begin{bmatrix} 20 + 2 & 0 + -6 \\ 10 + 6 & 0 + -18 \end{bmatrix}$ ✓
 $= \begin{bmatrix} 22 & -6 \\ 16 & -18 \end{bmatrix}$ ✓

④ ✓

c) $AB \neq BA$

$$BA = \begin{bmatrix} 5 & 0 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \times 4 + 0 \times 2 & 5 \times -1 + 0 \times -3 \\ -2 \times 4 + 6 \times 2 & -2 \times -1 + 6 \times -3 \end{bmatrix}$$

$$= \begin{bmatrix} 20 + 0 & -5 + 0 \\ -8 + 12 & 2 + -18 \end{bmatrix} \checkmark$$

$$= \begin{bmatrix} 20 & -5 \\ 4 & -16 \end{bmatrix} \checkmark \neq AB. \checkmark$$

5 ✓

d) Inverse matrix A.

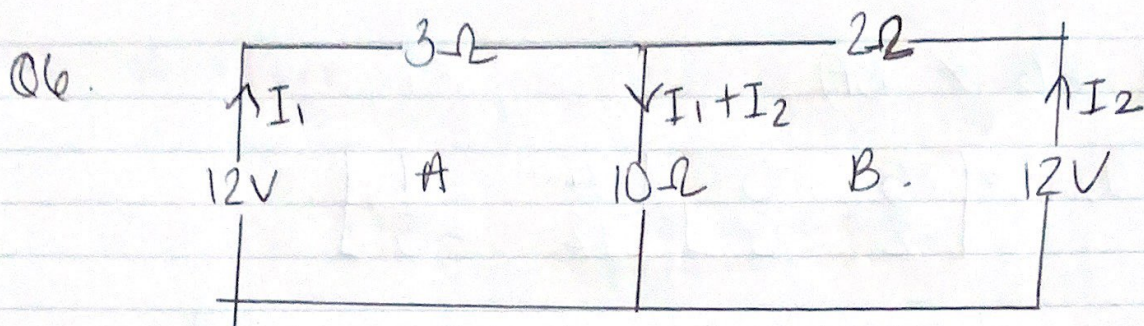
$$A = \begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix}$$

$|A| = (4 \times -3) - (2 \times -1) = -12 - -2 = -10$
Rearrange. $\begin{bmatrix} -3 & 1 \\ -2 & 4 \end{bmatrix}$ ✓

$$-\frac{1}{10} \begin{bmatrix} -3 & 1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{2}{10} & -\frac{4}{10} \end{bmatrix} \checkmark$$

3 ✓

$$= \begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$



$$12 = 3I_1 + 10(I_1 + I_2) \quad \checkmark$$

$$12 = 3I_1 + 10I_1 + 10I_2 \quad \checkmark$$

$$12 = 13I_1 + 10I_2 \quad \checkmark$$

$$12 = 2I_2 + 10(I_1 + I_2) \quad \checkmark$$

$$12 = 2I_2 + 10I_1 + 10I_2 \quad \checkmark$$

$$12 = 10I_1 + 12I_2 \quad \checkmark$$

$$13I_1 + 10I_2 = 12 \quad \checkmark$$

$$10I_1 + 12I_2 = 12 \quad \checkmark$$

$$\begin{bmatrix} 13 & 10 \\ 10 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \end{bmatrix} \quad \checkmark$$

$$I_1 = \frac{\begin{vmatrix} 12 & 10 \\ 12 & 12 \end{vmatrix}}{\begin{vmatrix} 13 & 10 \\ 10 & 12 \end{vmatrix}} \quad \checkmark$$

$$I_2 = \frac{\begin{vmatrix} 13 & 12 \\ 10 & 12 \end{vmatrix}}{\begin{vmatrix} 13 & 10 \\ 10 & 12 \end{vmatrix}} \quad \checkmark$$

$$I_1 = \frac{12 \times 12 - 12 \times 10}{13 \times 12 - 10 \times 10} \quad \checkmark$$

$$I_2 = \frac{13 \times 12 - 10 \times 12}{13 \times 12 - 10 \times 10} \quad \checkmark$$

$$I_1 = \frac{144 - 120}{156 - 100} \quad \checkmark$$

$$I_2 = \frac{156 - 120}{156 - 100} \quad \checkmark$$

$$I_1 = \frac{24}{56} \quad \checkmark \quad I_2 = \frac{36}{56} \quad \checkmark$$

$$I_1 \text{ through } 3\Omega \text{ is } \frac{24}{56} = \frac{3}{7} \text{ A} \quad \checkmark$$

$$I_2 \text{ through } 2\Omega \text{ is } \frac{36}{56} = \frac{9}{14} \text{ A} \quad \checkmark$$

$$I_1 + I_2 \text{ through } 10\Omega \text{ is } \frac{24}{56} + \frac{36}{56} = \frac{60}{56} = \frac{15}{14} \text{ A} \quad \checkmark$$

Q7. a) (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{3 - 5}{-2 - 4} = \frac{-2}{-6} = \frac{1}{3} \quad \checkmark$$

substitute $x = -2$ and $y = 3$
into $y = mx + c$.

$$y = mx + c$$

$$y = \frac{1}{3}x + c \quad \checkmark$$

$$3 = \frac{1}{3}x - 2 + c \quad \checkmark$$

$$3 = -\frac{2}{3} + c \quad \checkmark$$

$$\frac{11}{3} = c \quad \checkmark$$

$$\underline{y = \frac{1}{3}x + \frac{11}{3}} \quad \checkmark$$

15

5

b). line perpendicular to (a)
through (2,3)

$$\left[(a) m = \frac{1}{3} \right] \quad \underline{m = -3} \quad \checkmark$$

$$y = -3x + c \quad \checkmark$$

substitute when $x=2$ and $y=3$

$$y = -3x + c \quad \checkmark$$

$$3 = -3 \times 2 + c \quad \checkmark$$

$$3 = -6 + c$$

$$9 = c \quad \checkmark$$

$$\underline{\underline{y = -3x + 9}} \quad \checkmark$$

(5) $\checkmark$

Q8. $a + (n-1)d$.

$$a = £1.50$$

$$n = 50$$

$$d = £0.70$$

$$1.5 + (50-1) \times 0.7$$

$$= \underline{\underline{£35.80}} \quad \checkmark$$

(4) $\checkmark$

Q9. 5 speeds 10 rev/min to 700 rev/min

$$ar^{n-1} \quad a=10 \\ n=5$$

$$n=1 = 10(4\sqrt[4]{70})^{1-1} = \underline{10 \text{ rev/min}}$$

$$n=2 = 10(4\sqrt[4]{70})^{2-1} = \underline{29 \text{ rev/min}}$$

$$n=3 = 10(4\sqrt[4]{70})^{3-1} = \underline{84 \text{ rev/min}}$$

$$n=4 = 10(4\sqrt[4]{70})^{4-1} = \underline{242 \text{ rev/min}}$$

$$n=5 = 10(4\sqrt[4]{70})^{5-1} = \underline{700 \text{ rev/min}}$$

$$ar^{n-1} \\ 10r^{5-1} = 700 \\ r^4 = \frac{700}{10} \checkmark$$

$$r^4 = \frac{70}{1} \checkmark \\ r = \sqrt[4]{70} \checkmark$$

⑧ ✓